

Fourth Semester B.E. Degree Examination, June-July 2009
Engineering Mathematics – IV

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART – A

- 1 a. Solve $\frac{dy}{dx} = 2y + 3e^x$, $y(0) = 0$. Using Taylor's series method an find $y(0.1)$, $y(0.2)$. (06 Marks)
- b. Use Runge-Kutta method of fourth order to solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ at $x = 0.2$ with step length $h = 0.2$. (07 Marks)
- c. Use Milne's predictor –corrector method to find y at $x = 0.8$, given $\frac{dy}{dx} = x - y^2$ with,

X	0	0.2	0.4	0.6
Y	0	0.02	0.0795	0.1762

Apply corrector once. (07 Marks)

- 2 a. Find the analytic function $f(z) = u + iv$ if $v = e^x (x \sin y + y \cos y)$. (06 Marks)
- b. Find the image of lines parallel to $x -$ axis and lines parallel to $y -$ axis under the transformation $w = z^2$. Draw neat sketch. (07 Marks)
- c. Find the bilinear transformation that maps the points $z = -1, j, 1$ on to the points $w = 1, j, -1$. (07 Marks)

- 3 a. If $f(z)$ is analytic within and on a simple closed curve C and 'a' is a point within 'C' then prove that $f(a) = \frac{1}{2\pi j} \int_C \frac{f(z)}{z-a} dz$. (06 Marks)

- b. State Cauchy's residue theorem. Hence or otherwise evaluate –

$$\int_C \frac{e^{2z}}{(z+2)(z+4)(z+7)} dz \text{ for 'C' as } |Z|=3. \quad (07 \text{ Marks})$$

- c. Find the Taylor's series expansion of $f(z) = \frac{1}{(z+1)^2}$ about the point $z = -i$. (07 Marks)

- 4 a. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$. (06 Marks)

- b. Express polynomial $2x^3 - x^2 - 3x + 2$ in terms of Legendre polynomials. (07 Marks)
- c. Compute P_0, P_1, P_2, P_3, P_4 using Rodrigue's formula. (07 Marks)

PART – B

- 5 a. Fit a parabola $y = a + bx + cx^2$, given the data : (06 Marks)

x	-3	-2	-1	0	1	2	3
y	4.63	2.11	0.67	0.09	0.63	2.15	4.58

- b. Obtain the coefficient of correlation and the liens of regression if : (07 Marks)

x	1	3	4	2	5	8	9	10	13	15
y	8	6	10	8	12	16	16	10	32	32

- c. A tea set has four sets of cups and saucers. Two of these sets are of one colour and the other two sets are of different colours. (totally three colours). If the cups are placed randomly on saucers, what is the probability that no cup is on a saucer of same colour. (07 Marks)

- 6 a. Define i) Random variable ii) Discrete probability distribution with an example. (06 Marks)
- b. The probability that a man aged 60 will live up to 70 is 0.65. What is the probability that out of 10 men, now aged 60, i) exactly 9, ii) at the most 9 iii) at least 7, will live up to the age of 70 years. (07 Marks)
- c. In a normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation, given that $A(0.5) = 0.19$ and $A(1.4) = 0.42$. (07 Marks)
- 7 a. Find the probability that in 100 tosses of a fair coin between 45% and 55% of the outcomes are heads. (06 Marks)
- b. A mechanist is making engine parts with axle diameter of 0.7 inches. A random sample of 10 parts showed a mean of 0.472 inches with a standard deviation of 0.04 inches. On the basis of this sample, can it be concluded that the work is inferior at 5% level of significance. (07 Marks)
- c. For the following data test the hypothesis that the accidents are uniformly distributed over all the days of the week for 99% confidence.

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Total
No. of accidents	14	16	8	12	11	9	14	84

(07 Marks)

- 8 a. Find the –
Marginal distribution of x
Marginal distribution of y
Cov (x, y) if the joint pdf of x and y is

x \ y	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

- b. Find the fixed probability vector of regular stochastic matrix (06 Marks)

$$A = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0 & 1 & 0 \end{bmatrix}$$

(07 Marks)

- c. A company executive changes his car every year. If he has a car of make A, he changes over to make B. from make B he changes over to make C. if he has car 'C' then he gives equal preference to change over to make A or make B car. If he had a car of make C in year 2008 find the probability that he will have a car of i) make A in 2010, ii) make 'C' in 2010. (07 Marks)

Fourth Semester B.E. Degree Examination, June-July 2009
Microcontrollers

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions choosing at least two questions from each unit.

PART – A

- 1 a. Define microcontroller and differentiate the RISC and CISC processors. (05 Marks)
 b. With the neat block diagram, explain the architecture of 8051. (10 Marks)
 c. Show the neat schematic interface 8K External Data RAM to 8051. (05 Marks)
- 2 a. What is addressing mode? Explain different addressing modes with examples. (09 Marks)
 b. Specify the memory area for bit level logical instructions used in 8051 and list bit level logical instructions. (05 Marks)
 c. Write an Assembly language program to add two input data's of 16-bit result in three different addressing modes. (06 Marks)
- 3 a. Explain the following instructions with their function byte and cycle used :
 i) CJNE dest, source target ; ii) A call target
 iii) DJNZ R₁,rel ; iv) SWAP A v) DA A (10 Marks)
 b. Explain the different types of Jump instructions in 8051. (06 Marks)
 c. What is interrupt? List different interrupts using 8051 with their ISR address. (04 Marks)
- 4 a. Write a C-program to toggle all bits of P₀ and P₂ continuously with 250m sec delay. Use inverting operator. (08 Marks)
 b. What is data serialization? Explain different types with examples. (06 Marks)
 c. Write a 8051 C-program to convert a given hex-data OFFls in to its equivalent decimal data and display the result digits on P₀, P₁ and P₂ (06 Marks)

PART – B

- 5 a. Explain T mod and T con registers with its bit pattern. (08 Marks)
 b. Explain mode-2 programming with neat sketch and specify the program steps. (06 Marks)
 c. Assuming that clock pulses are Fed in to Pty T₁, write a program for counter-1 in mode-2 to count pulses and display the state of TL₁ count on P₂ (06 Marks)
- 6 a. Explain RS-323 hand shaking signals and specify the purpose of max-232 while interfacing. (07 Marks)
 b. Write 8051 program to transfer serially the message "VTU BELGAUM" continuously at a band rate of 9600. (07 Marks)
 c. Explain the importance of TI and RI Flags. (06 Marks)
- 7 a. Explain IE and IP registers with their bit pattern and show how priorities change with example. (10 Marks)
 b. Write 8051 interrupt program to do the following :
 i) Receive data serially P₂ and sent it to P₁ continuously.
 ii) Make timer-0 to generate a square wave of 5KH_z frequency at port Po.1.
 Assume XTAL – 11.059MH_z at a band of 9600. (10 Marks)
- 8 a. Explain the registers and pins of LCD and write an ALP to display message "HELLO" as LCD displays. (10 Marks)
 b. Describe the 8051 connection to stepper motor and write an Assembly language program to rotate the motor clockwise for 180°. Assuming motor specification 1.8°/step (10 Marks)

Fourth Semester B.E. Degree Examination, June-July 2009
Control Systems

Time: 3 hrs.

Max. Marks:100

Note:1. Answer any FIVE full questions, choosing at least two questions from each.
2. Missing data may be suitably assumed.

Part-A

- 1 a. Mention the merits and demerits of open loop and closed loop control systems and give an example for each. (06 Marks)
- b. For the mechanical system shown in figure Q1 (b), obtain the force-voltage analogous network. (08 Marks)

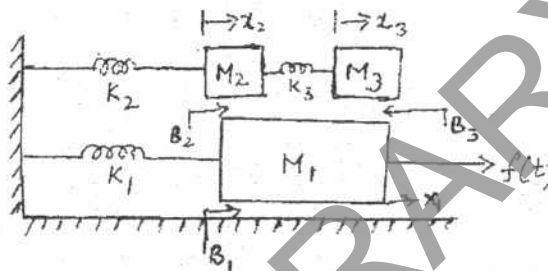


Fig. Q1 (b)

- c. Obtain the transfer function of an armature controlled dc servomotor. (06 Marks)
- 2 a. Explain briefly the following terms:
 - i) Forward path. ii) Path gain. iii) Loop gain. iv) Canonical form. (08 Marks)
 - b. Obtain the C/R ratio for the block diagram shown using block-diagram reduction technique. (06 Marks)

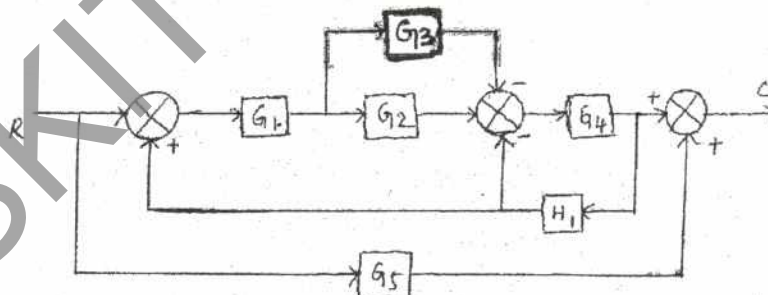


Fig. Q2 (b)

- c. Find $\frac{C(s)}{R(s)}$ by Mason's gain formula (Fig. Q2 (c)). (06 Marks)

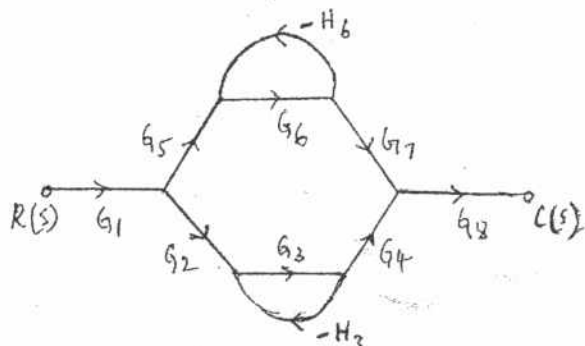


Fig. Q2 (c)
1 of 3

- 3 a. Derive an expression for the under damped response of a second order feed back control system for step input. (06 Marks)
- b. Obtain expressions for rise-time and peak-time for a second-order feed back system response for a step input (under-damped case). (06 Marks)
- c. A positional servomechanism is characterized by an open loop transfer function

$$G(s) = \frac{k}{s(s+\alpha)}, \text{ where } k \text{ and } \alpha \text{ are positive constants, for a unity feedback. Find the values of } k \text{ and } \alpha \text{ for a damping coefficient value of } 0.6 \text{ and damped frequency of } 8 \text{ rad/sec. Also find the peak value of the response when the system is excited by a step of } 2 \text{ volts. (08 Marks)}$$

- 4 a. Find K_P , K_V and K_a for the system whose open loop transfer function is given by,

$$G(s)H(s) = \frac{10(s+2)(s+3)}{s(s+1)(s+5)(s+4)}$$

Also find the steady-state error for an input $r(t) = 3 + t + t^2$. (08 Marks)

- b. Explain Routh-Hurwitz 's' criterion for determining the stability of a system and mention by limitations. (04 Marks)
- c. A unity feedback control system has:

$$G(s) = \frac{k(s+13)}{s(s+3)(s+7)}$$

Using the Routh's criterion, calculate the range of 'k' for which the system is

- i) Stable
ii) Has its closed loop poles more negative than -1 . (08 Marks)

Part-B

- 5 a. Explain briefly the following terms with respect to root-locus technique:

- i) Centroid.
ii) Asymptote.
iii) Break away point. (06 Marks)

- b. Sketch the root locus plot for a closed loop system having an open-loop transfer function:

$$G(s)H(s) = \frac{k(s+2)}{s(s+1)} \text{ for all values of } k \text{ from } 0 \text{ to } \infty. \text{ Comment on the stability of the system. (08 Marks)}$$

- c. Show that a part of the root-locus for the open loop transfer function:

$$G(s)H(s) = \frac{k(s+2)}{s(s+1)} \text{ is a circle. (06 Marks)}$$

- 6 a. The open loop transfer function of a negative feedback control system is:

$$GH(s) = \frac{1}{s(s+1)(s+\frac{1}{2})}$$

Sketch the polar plot and hence find the following:

- i) Phase cross-over frequency.
ii) Gain cross-over frequency.
iii) Gain-margin.
iv) Phase-margin. (10 Marks)

- b. Explain in detail the procedural steps of Nyquist stability criterion. (04 Marks)

- c. A feed back control system has loop transfer function: $GH(s) = \frac{1}{s(s+1)}$

Sketch the Nyquist plot and comment on the stability of a system. (06 Marks)

- 7 a. Derive expressions for resonant peak and resonant frequency for a second order system. (06 Marks)
- b. Find the open-loop transfer function for a unity feed back second order control system for which resonant peak is 1.1 units and resonant frequency is 11.2 radians/sec. (06 Marks)
- c. Sketch the Bode-plot for the transfer function: $G(s) = \frac{Ks^2}{(1+0.2s)(1+0.02s)}$
Determine the value of 'k' for the gain cross over frequency to be 5 rad/sec. (08 Marks)
- 8 a. Compare transfer function approach and state variable approach of analyzing control system. (04 Marks)
- b. A feed back system is characterized by the closed loop transfer function:

$$GH(s) = \frac{s^2 + 3s + 3}{s^3 + 2s^2 + 3s + 1}$$
 Obtain its state model. (08 Marks)
- c. State the properties of state transition matrix. Obtain the state transition matrix for:

$$A = \begin{bmatrix} 0 & -1 \\ 2 & -3 \end{bmatrix}$$
 (08 Marks)

Fourth Semester B.E. Degree Examination, June-July 2009
Signals and Systems

Time: 3 hrs.

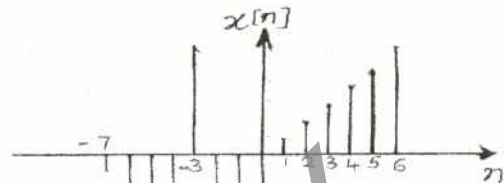
Max. Marks:100

Note: Answer any FIVE full questions, selecting at least TWO questions from each part.

PART - A

1 a. A function $x[n]$ is defined by

$$x[n] = \begin{cases} -(n+8) & \text{for } -8 < n < -3 \\ 6 & \text{for } n = -3 \\ -6 & \text{for } -3 < n < 0 \\ n & \text{for } -1 < n < 7 \\ 0 & \text{otherwise} \end{cases}$$



Sketch $y[n] = 3 \cdot x[n/2 + 1]$

(04 Marks)

b. Perform the following operations (addition & multiplication) on given signals. Fig.1(b).

(i) $y_1(t) = x_1(t) + x_2(t)$ (ii) $y_2(t) = x_1(t) \cdot x_2(t)$



Fig.1(b)

(04 Marks)

c. Distinguish between i) Energy signal & power signal ii) Even & odd signal.

(06 Marks)

d. Explain the following properties of systems with suitable example:

i) Time invariance ii) Stability iii) Linearity.

(06 Marks)

2 a. Find the convolution integral of $x(t)$ & $h(t)$ and sketch the convolved signal:

$$x(t) = \delta(t) + 2\delta(t-1) + \delta(t-2), \quad h(t) = 3, \quad -3 \leq t \leq 2.$$

(08 Marks)

b. Determine the convolution sum of the given sequence

$$x(n) = \{3, 5, -2, 4\} \text{ and } h(n) = \{3, 1, 3\}$$

(06 Marks)

c. Show that i) $x(t) * \delta(t-t_0) = x(t-t_0)$

$$\text{ii) } x(n) = \sum_{k=-\infty}^{\infty} x(k) \cdot \delta(n-k)$$

(06 Marks)

3 a. The impulse response of the system is $h(t) = e^{-4t} u(t-2)$. Check whether the system is stable, causal and memoryless.

(06 Marks)

b. Draw the direct form-I & direct form-II implementation of the following difference

$$\text{equation. } y(n) - \frac{1}{4}y(n-1) + y(n-2) = 5x(n) - 5x(n-2)$$

(06 Marks)

c. Find the forced response of the system shown in Fig.3(c), where $x(t) = \text{const.}$

(08 Marks)

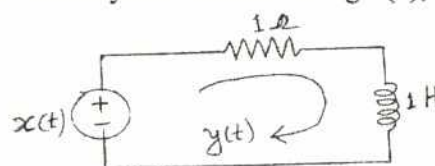


Fig.3(c)

- 4 a. State the condition for the Fourier series to exist. Also prove the convergence condition [Absolute integrability]. (06 Marks)
- b. Prove the following properties of Fourier series. i) Convolution property ii) Parsevals relationship. (06 Marks)
- c. Find the DTFS harmonic function of $x(n) = A \cos(2\pi n/N_0)$. Plot the magnitude and phase spectra. (08 Marks)

PART - B

- 5 a. State and prove the following properties of Fourier transform.
i) Time shifting property ii) Differentiation in time property iii) Frequency shifting property. (09 Marks)
- b. Plot the Magnitude and phase spectrum of $x(t) = e^{-|t|}$ (06 Marks)
- c. Determine the time domain expression of $X(j\omega) = \frac{2j\omega + 1}{(j\omega + 2)^2}$ (05 Marks)
- 6 a. The spectrum $X(j\omega)$ of signal is shown in Fig.6(a). Draw the spectrum of the sampled signal at i) half the Nyquist rate ii) Nyquist rate and iii) Twice the Nyquist rate. Mark the frequency values clearly in the figure. (12 Marks)

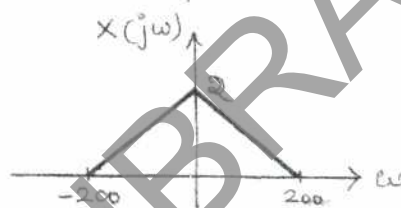


Fig.6(a)

- b. Define and explain Nyquist sampling theorem with relevant figures. Give significance of this theorem. (08 Marks)
- 7 a. Describe the properties of Region of convergence and sketch the ROC of two sided sequences, right sided sequence and left sided sequence. (10 Marks)
- b. Find the inverse Z-transform of $X(z) = \frac{1}{(z^2 - 2z + 1)(z^2 - z + \frac{1}{2})}$ using partial fraction method. (10 Marks)
- 8 a. Solve the difference equation $y(n+2) - \frac{3}{2}y(n+1) + \frac{1}{2}y(n) = \left(\frac{1}{4}\right)^n$ for $n \geq 0$ with initial conditions $y(0) = 10$ and $y(1) = 4$. Use z-transform. (12 Marks)
- b. Explain how causality and stability is determined in terms of z-transform. Explain the procedure to evaluate Fourier transform from pole zero plot of z-transform. (08 Marks)

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Fourth Semester B.E. Degree Examination, June-July 2009
Fundamentals of HDL

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer any FIVE full questions selecting at least TWO from each part.

PART – A

- 1 a. Write the result of all shift and rotate operations in VHDL after applying them to a 7 bit vector A = 1001010. (06 Marks)
- b. Explain composite and Access data types with an example for each. (08 Marks)
- c. Mention different styles (types) of descriptions. Explain mixed type and mixed language descriptions. (06 Marks)
- 2 a. How do you assign delay to a signal assignment statement? Explain with an example in VHDL and verilog. (04 Marks)
- b. What is a vector? Give an example for VHDL and verilog vector data types. (04 Marks)
- c. With the help of a truth table and K-maps write Boolean expression for a 2-bit magnitude comparator. Write VHDL / verilog code (12 Marks)
- 3 a. Write VHDL code for a D-latch using variable assignment and signal assignment statements. With simulation waveforms clearly distinguish between the 2 statements. (10 Marks)
- b. Explain verilog Repeat and Forever statements with an example. (04 Marks)
- c. Write verilog code for a 4-bit counter with synchronous hold. (06 Marks)
- 4 a. What is binding? Discuss binding between two modules in verilog. (06 Marks)
- b. Write VHDL behavioral description of a tristate buffer. Use this as a component for structural description of a 2 to 4 decoder with tristate output. (10 Marks)
- c. Explain the use of Generic (in VHDL) and parameter (in verilog) with an example. (04 Marks)

PART – B

- 5 a. Write verilog code to convert a signed binary to Integer using task. (08 Marks)
 - b. Write VHDL / verilog function to find greater of 2 signed numbers. (04 Marks)
 - c. Table Q.5(c) below shows a file containing real numbers. Write a VHDL code for reading the file, multiply the first number by 2, second by 5, third by 3 and fourth by 4. The products to be stored in real variables z, z1, z2 and z3 respectively. (08 Marks)
- | | | |
|--------|--------|------|
| -13.4 | -5.564 | 0.23 |
| -55.32 | | |
- Table Q.5(c) File file _real . txt
- 6 a. Write a note on packages in VHDL. (05 Marks)
 - b. Write VHDL code for addition of two 5 x 5 matrices using a package. (07 Marks)
 - c. Write the block diagram and function table of a SRAM. Using these, write a verilog description for 16 x 8 SRAM. (08 Marks)
 - 7 a. How to invoke a verilog module from a VHDL module? Explain with an example of a mixed language description for a full adder using 2 half adders. (10 Marks)
 - b. Write a mixed language description of a 9-bit adder consisting of three 3-bit carry-look ahead adders to show how a verilog module invokes VHDL entity. (10 Marks)
 - 8 a. Explain extraction of synthesis information from Entity. (04 Marks)
 - b. With an example explain verilog synthesis information extraction from module inputs and outputs. (04 Marks)
 - c. Write VHDL / verilog code for signal assignment statement $Y = (2 * x + 3)$ for an entity with one input X of 2-bits and one output Y of 4-bits. Show mapping of this signal assignment to gate level. (12 Marks)

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Fourth Semester B.E. Degree Examination, June-July 2009
Linear ICs and Applications

Time: 3 hrs.

Max. Marks:100

Note: 1. Answer any FIVE full questions selecting at least TWO from each part.

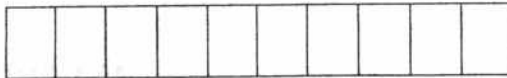
PART - A

1. a. With a neat circuit diagramed explain basic operational amplifier circuit. (06 Marks)
 b. Explain potential divider bias for an op-amp input, with the necessary design steps. (06 Marks)
 c. i) With a neat circuit diagram, explain direct-coupled inverting amplifier.
 ii) Also, design an inverting amplifier using a 741 op-amp, for which voltage gain is to be 66 and the output voltage amplitude is to be 3V. Given : $I_{B(max)} = 500 \text{ nA}$. (08 Marks)
2. a. Explain the operation of a high input impedance capacitor-coupled voltage follower, with a neat circuit diagram. Obtain the expression for input impedance of the circuit. (08 Marks)
 b. Briefly discuss the upper cutoff frequency of an op-amp circuit. Show how the cutoff frequency can be set for inverting and noninverting amplifiers. (06 Marks)
 c. Design a capacitor-coupled inverting amplifier to operate with a + 20V supply. The minimum input signal level is 50mV, the voltage gain is to be 68, the load resistance is 500Ω , and the lower cutoff frequency is to be 200Hz. Use 741 op-amp. (06 Marks)
3. a. Define and briefly explain:
 i) Loop gain
 ii) Loop phase shift
 iii) Phase margin
 iv) Unity gain bandwidth. (08 Marks)
 b. With a neat circuit diagram, explain Zin Mod method of frequency compensation. Write the equation for the feedback factor. (08 Marks)
 c. Calculate the slew rate-limited cutoff frequency for a voltage follower circuit using a 741 op-amp, if the peak of sine wave output is to be 6V. Determine the maximum peak value of the sinusoidal output voltage that will allow the 741-voltage follower. Circuit to operate at the 800 kHz. Unity gain cutoff frequency. Given : $S = 0.5 \text{ v}/\mu\text{s}$. (04 Marks)
4. a. With a neat circuit diagram, explain the operation of high input impedance full-wave precision rectifier. Draw the voltage waveforms at various points and write the appropriate equations to show that full-wave rectification is performed. (12 Marks)
 b. Design an instrumentation amplifier to have an overall voltage gain of 625. The input signal amplitude is 10mV, 741 op-amps are to be used, and the supply is $\pm 20\text{V}$. (08 Marks)

PART - B

5. a. With a neat circuit diagram and waveforms, explain the operation of triangular/rectangular wave generator. (08 Marks)
 b. Explain working of an Wein bridge oscillator with the help of circuit diagram, waveforms and equations. (06 Marks)
 c. Using a 741 op-amp with a supply of $\pm 15\text{V}$, design a phase-shift oscillator to have an output frequency of 5.5KHz. Given : $A_v = 29$. (06 Marks)

- 6 a. With a neat circuit diagram and waveforms, explain the operation of inverting Schmitt trigger. (06 Marks)
- b. Using a 741 op-amp, design a second order high-pass filter to have a cutoff frequency of 15KHz. (06 Marks)
- c. A capacitor-coupled zero crossing detector is to handle a 2KHz square wave with a peak-to-peak amplitude of 10V. Design a circuit using a 741 op-amp with a $\pm 15V$ supply. Estimate the minimum op-amp slew rate to give a reasonably undistorted output. Also, calculate the lowest sine wave input frequency that can be applied without the phase shift error exceeding 3° . Given : $V_B = 0.1 V$, $I_{B(max)} = 500 nA$. (08 Marks)
- 7 a. With a neat schematic, explain the salient features of a 723 regulator. (08 Marks)
- b. Explain the terms line regulation, load regulation and ripple rejection for a dc voltage regulator. (06 Marks)
- c. What is the principle of switch-mode power supplies? Discuss its advantages and disadvantages. (06 Marks)
- 8 Explain the following with neat diagrams and waveforms:
- a. 555 timer as a stable multivibrator
- b. 566 voltage controlled oscillator
- c. R-2R ladder DAC
- d. Dual-slope ADC. (20 Marks)



Fourth Semester B.E. Degree Examination, June-July 2009
Advanced Mathematics - II

Time: 3 hrs.

Max. Marks:100

Note: Answer any FIVE full questions

- 1 a. If l, m, n and l', m', n' are the direction cosines of the lines OP & OQ and θ be the angle between them then show that $\cos\theta = ll' + mm' + nn'$. Also derive the condition for the perpendicularity of OP & OQ. (07 Marks)
- b. Find the equation of the plane which passes through the point $(3, -3, 1)$ and is
 i) Parallel to the plane $2x + 3y + 5z + 6 = 0$.
 ii) Perpendicular to the planes $7x + y + 2z = 6$ and $3x + 5y - 6z = 8$. (07 Marks)
- c. Find the equations to the two planes which bisect the angles between the planes $3x - 4y + 5z = 3$ and $5x + 3y - 4z = 9$. (06 Marks)
- 2 a. Obtain the equation of a plane passing through the line of intersection of the planes $7x - 4y + 7z + 16 = 0$ and $4x + 3y - 2z + 13 = 0$ and perpendicular to the plane $x - y - 2z + 5 = 0$. (07 Marks)
- b. Find the equation of the two straight lines through the origin each of which intersects the straight line $\frac{1}{2}(x-3) = y-3 = z$ and it is inclined at an angle of 60° to it. (07 Marks)
- c. Find the magnitude and the equation of the shortest distance between the lines $\frac{x}{2} = \frac{y}{-3} = \frac{z}{1}$ and $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$. (06 Marks)
- 3 a. If $\vec{A} = i + 2j + 3k$, $\vec{B} = -i + 2j + k$, $\vec{C} = 3i + j$ Find P such that $\vec{A} + \vec{PB}$ is perpendicular to \vec{C} . Also, find the dot product of A with B and $A \cdot (B + C)$. (07 Marks)
- b. If $\vec{A} = 4i + 3j + k$, $\vec{B} = 2i - j + 2k$ find a unit vector perpendicular to the plane containing both \vec{A} & \vec{B} . Also, show that A is not perpendicular to \vec{B} . (07 Marks)
- c. Find the constant a such that the vectors $\vec{AB} = ai - 5j + 2k$, $\vec{AD} = -7i + 14j - 3k$, $\vec{AC} = 11i + 4j + k$ are coplanar. (06 Marks)
- 4 a. A particle moves along a curve $x = e^{-t}$, $y = \cos 3t$, $z = 2\sin 3t$ where t is the time variable. Determine its velocity and acceleration vectors and also, find the magnitudes of velocity and acceleration at $t = 0$. (07 Marks)
- b. Find the directional derivative of $f(xyz) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the vector $I + 2J + 2K$. Also, find the direction along which it is maximum. (07 Marks)
- c. Prove that $\nabla r^n = 2r^{n-2} R$ where $R = xi + yj + zk$ and $r = |R|$. (06 Marks)
- 5 a. For what value of 'a' does this vector $\vec{P} = (ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ has zero divergence. Also find $\nabla \times \vec{P}$. (07 Marks)
- b. Show that $\text{curl grad } \phi = \vec{0}$ (07 Marks)
- c. Given that $\vec{F} = (x + y + 1)i + j - (x + y)k$. Show that $\vec{F} \cdot \text{curl } F = 0$ (06 Marks)

- 6 a. Using the definition show that $L\{t^n\} = \frac{n!}{s^{n+1}}$ (05 Marks)
- b. Find $L\{e^{-t} t \cos at\}$ (05 Marks)
- c. Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$ (05 Marks)
- d. Find $L\{\cos (at + b)\}$ (05 Marks)
- 7 a. Show that $L\{t^n\} = (-1)^n \frac{dn}{ds^n} f(s)$ where n is an integer. (05 Marks)
- b. Find $L^{-1}\left\{\frac{s^2 - 10s + 13}{(s - 7)(s^2 - 5s + 6)}\right\}$ (05 Marks)
- c. Find $L^{-1}\left\{\frac{s + 2}{s^2 - 4s + 13}\right\}$ (05 Marks)
- d. If $L\{f(t)\} = f(s)$, show that $L^{-1}\left\{\int_s^\infty f(s) ds\right\} = \frac{f(t)}{t}$ (05 Marks)
- 8 a. Solve the initial value problem using Laplace transforms $(D^3 - 3D^2 + 3D - 1)y = 0$ given that $y(0) = 1, y'(0) = 0, y''(0) = 0$ (10 Marks)
- b. Solve the simultaneous equations $x' - y = e^t, y' + x = \sin t, x(0) = 1, y(0) = 0$. (10 Marks)
